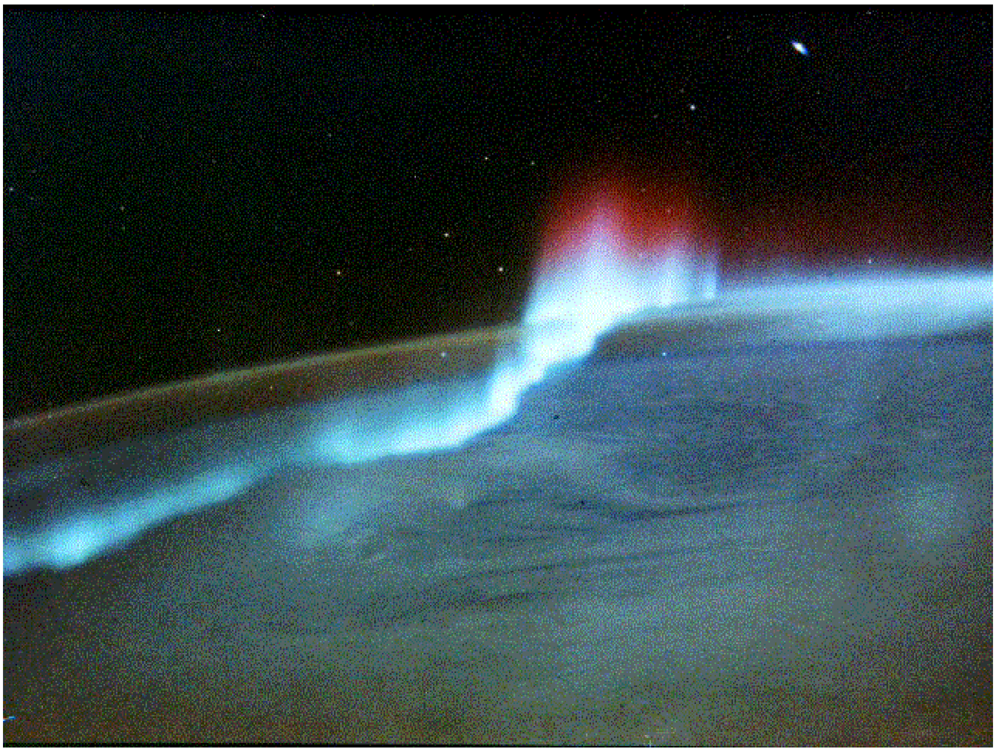
KTH Electrical Engineering

Auroral current sheets
Theory overview



Aurora as seen from the space shuttle. Photo: NASA.

T. Karlsson

Space and Plasma Physics
School of Electrical Engineering, KTH

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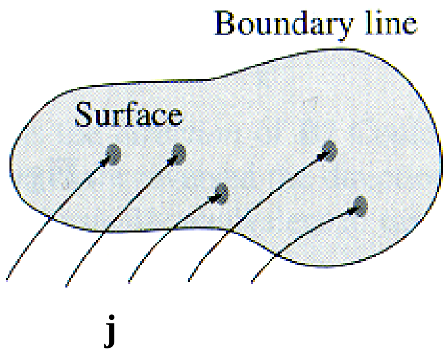
1. Current density

The current density **j** is a vector field with dimension [**j**] = Am^{-2}.



The total current I through a surface S is

$$\boxed{I = \int_S \mathbf{j} \cdot d\mathbf{S}} \tag{1.1}$$

2. The relationship between current and magnetic field: Ampère's law

The mathematical relationship between current density and magnetic field is given by (when no temporal variations are present)

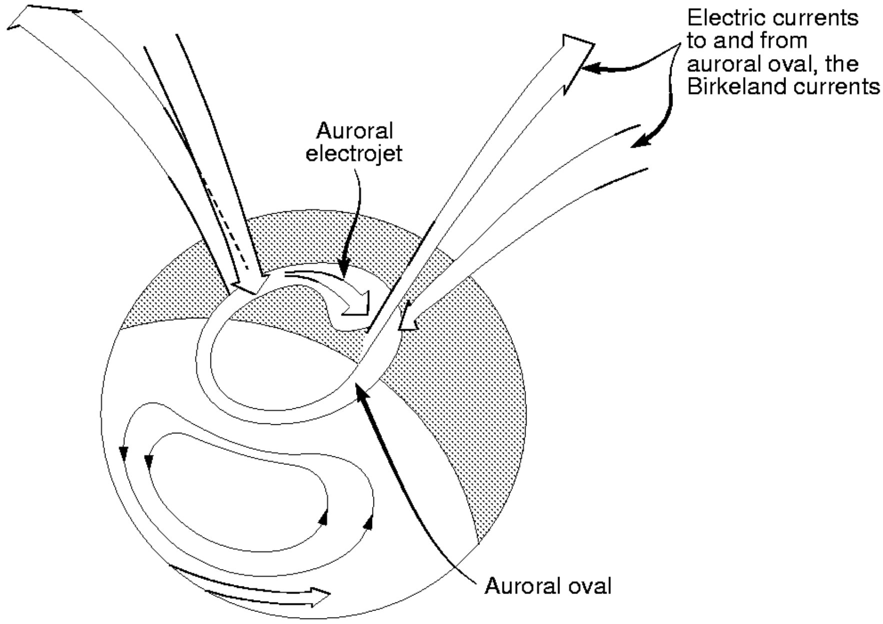
$$\boxed{\nabla \times \mathbf{B} = \mu_0 \mathbf{j}} \tag{2.1}$$

where $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}. In component form this is

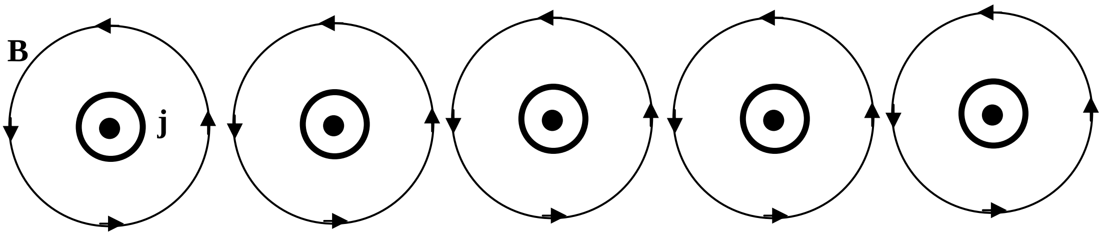
$$\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z},\ \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x},\ \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) = \mu_0 \left(j_x,\ j_y,\ j_z \right) \tag{2.2}$$

3. Current sheets

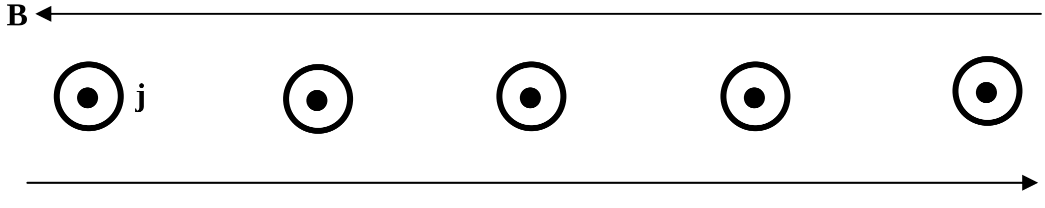
In the ionosphere and the magnetosphere the current is often flowing in extended two-dimensional current sheets, where the current is directed along the geomagnetic field. (Such currents are called “field-aligned currents” or “Birkeland currents”.)



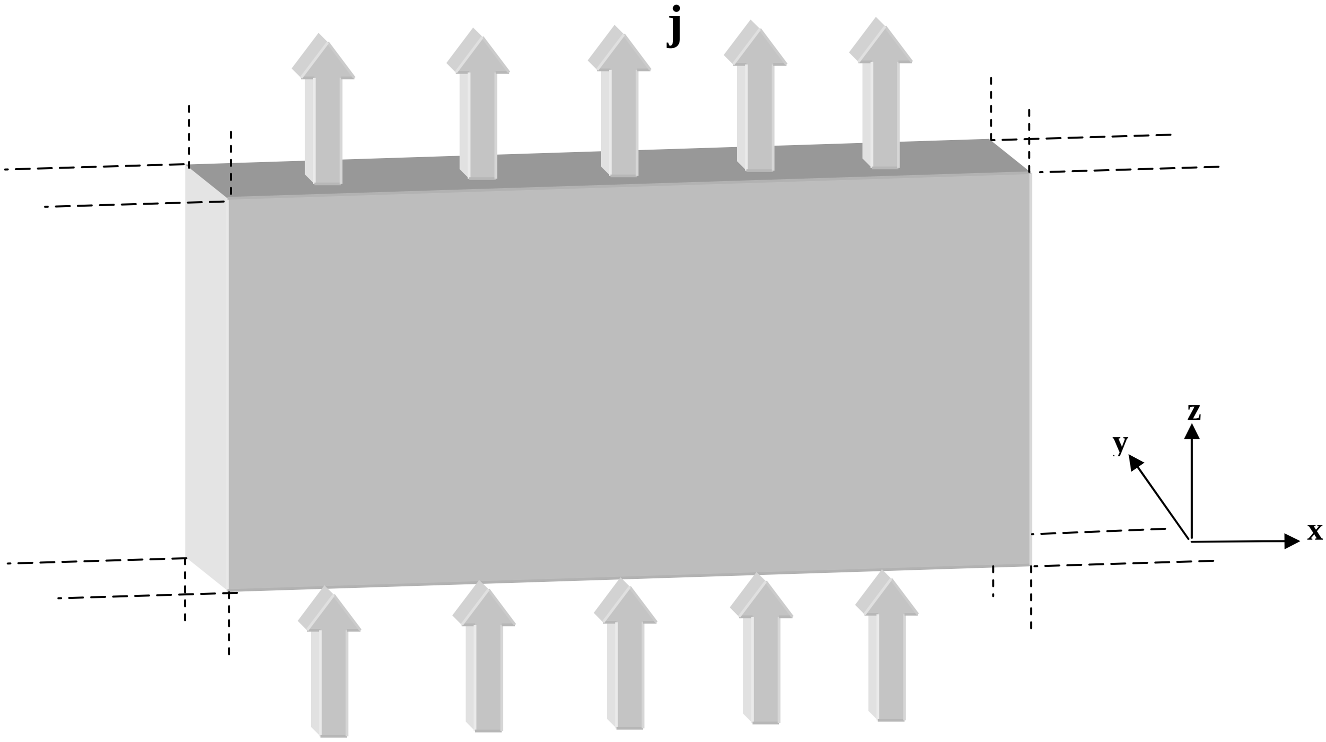
What will the magnetic signature of such current sheets be? We can get an idea of this by approximating the sheet by a number of line currents:



The magnetic field between the line currents cancel and the resulting field should be something similar to this:



A more mathematical treatment can be done as follows: consider a currents sheet with the current floating in the z-direction (see coordinate system in the figure) and extending infinitely in the x- and z-directions. (This is obviously an approximation, but often a good one for current sheets oriented along the auroral oval.)



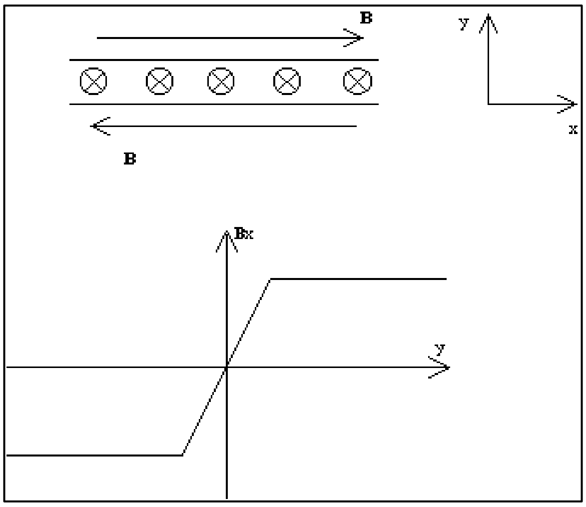
With the coordinate system as the figure, the infinite extension of the currents sheet means that all derivatives with respect to x and z are identically zero. Ampère's law is then simplified to:

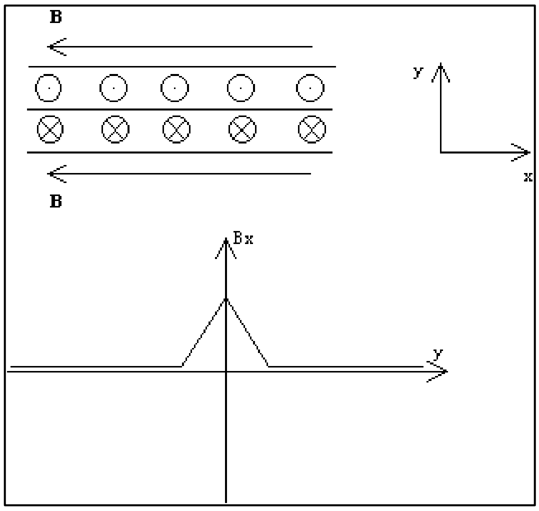
$$\left(\frac{\partial B_z}{\partial y},\ 0,\ -\frac{\partial B_x}{\partial y}\right) = \mu_0\left(0,\ 0,\ j_z\right) \tag{3.1}$$

The relationship between the magnetic field variation and the current density is thus reduced to the simple equation

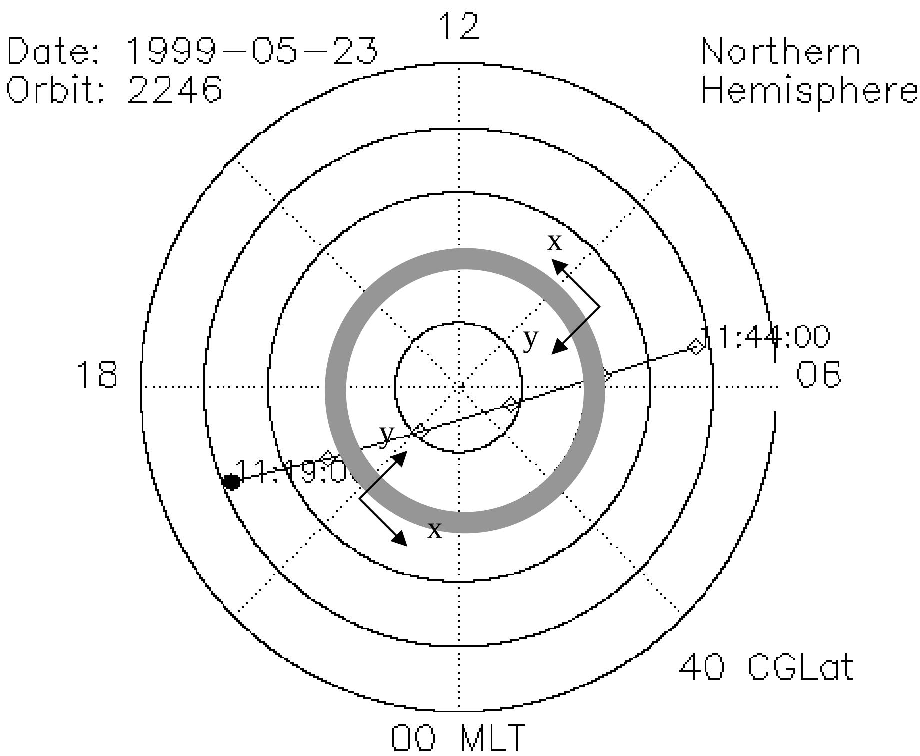
$$\boxed{j_z = -\frac{1}{\mu_0}\frac{\partial B_x}{\partial y}} \tag{3.2}$$

This is a very useful approximation when you calculate current densities from magnetic field data measured by satellites. It means that in regions of upward current ($j_z > 0$), the derivative in the y-direction of B_x is negative, and vice versa. Schematically then, the B-field signature for upward and downward currents will look like in the following figure:

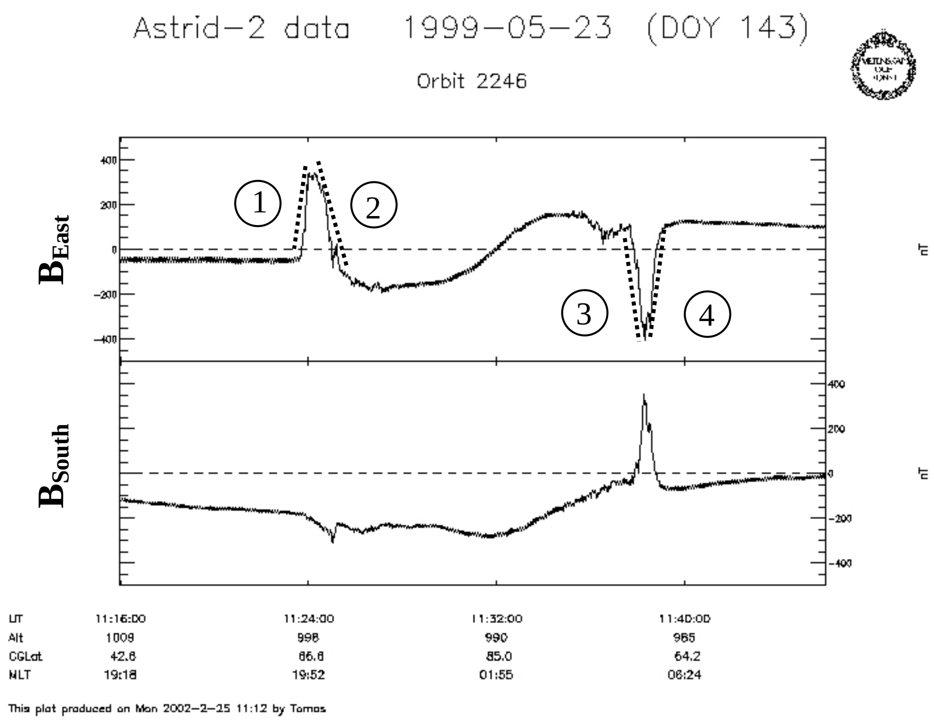




As an example, we can look at magnetic field data from the Astrid-2 satellite. Let us consider an orbit where Astrid-2 passed the auroral oval twice according to the figure below:



We expect that any currents associated with the aurora should be oriented along the auroral oval, and locally we can approximate the currents with infinite current sheets. If we let the x-direction correspond to East and y to North, we can calculate the current density down to or out from the ionosphere by studying the East component of the measured magnetic field. In the figure below, we can see four regions with a more or less constant derivative in the B_x component ($=B_{East}$) indicated with dotted lines and numbered from 1 to 4. (We will ignore the small-scale variations with are superimposed on the large-scale trend.)



The change in B_{East} around 11:32 UT is a mathematical artifact, which comes about since the definition of "East" changes as you pass the pole, and is not associated with a current. Now, with the help of Ampère's law we can e.g. determine if these four currents sheets correspond to currents out of ($j_z > 0$) or into ($j_z < 0$) the ionosphere. We summarize the results in a table:

1)	$\frac{\partial B_x}{\partial y} > 0$	$\Rightarrow$	$j_z < 0$	Into the ionosphere
2)	$\frac{\partial B_x}{\partial y} < 0$	$\Rightarrow$	$j_z > 0$	Out of the ionosphere
3)	$\frac{\partial B_x}{\partial y} > 0$	$\Rightarrow$	$j_z < 0$	Into the ionosphere
4)	$\frac{\partial B_x}{\partial y} < 0$	$\Rightarrow$	$j_z > 0$	Out of the ionosphere

Note that for current sheets 3 and 4 the satellite is moving southwards, i.e. in the negative y-direction, which you have to take into account when you decide the sign of $\frac{\partial B_x}{\partial y}$ from the figure. Also notice that for current sheets 1 and 2, the x-component of the magnetic field is much larger than the y-component, while 3 and 4 have a considerable y-component. This indicates that for the latter sheet out approximation that the sheet is oriented in the east-west direction is not totally correct.